Quantifying Information Modification in Cellular Automata using Pointwise Partial Information Decomposition

CCS 2019

Conor Finn
Joseph Lizier

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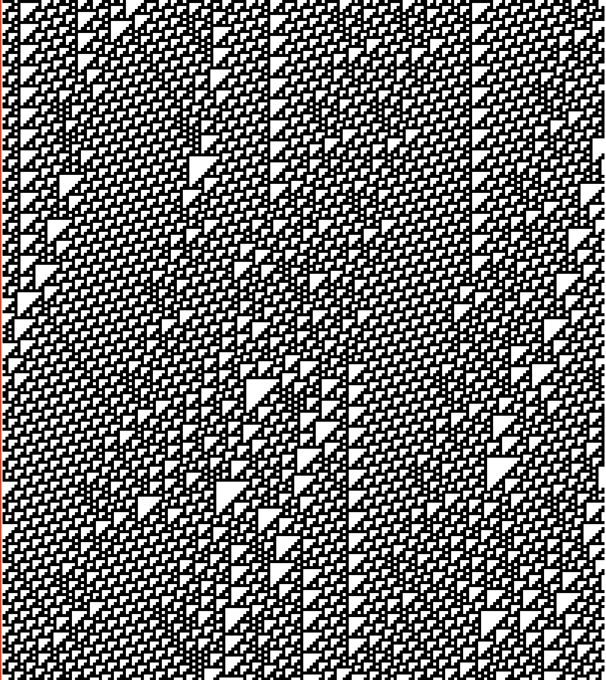
THE UNIVERSITY OF SYDNEY



How can we quantify intrinsic, emergent computation?

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- Turing described computation in terms of
  - information storage
  - information transfer
  - information modification
- Langton (1990) informally discusses emergent computation using this terminology
- Can we formalise these quantities as information-theoretic quantites?

  $\rightarrow$ **Information dynamics**

Information theory

- Mutual information

$$I(X;Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$
$$= \mathbb{E}_{X,Y}\left[i(x,y)\right]$$

- Pointwise mutual information

$$i(x;y) = \log \frac{p(x,y)}{p(x)p(y)}$$

- Joint mutual information

$$I(X;YZ) = \sum_{x,y,z} p(x,y,z) \log \frac{p(x,y,z)}{p(x)p(y,z)}$$

Information dynamics

- Use pointwise information theory to quantify
  - storage
  - transfer
  - modification
- Local in time and space

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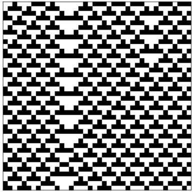
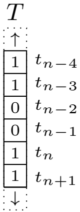
Rule 54





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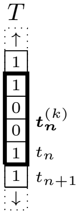

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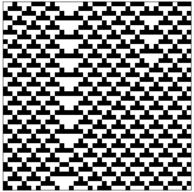

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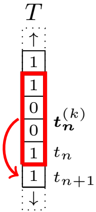

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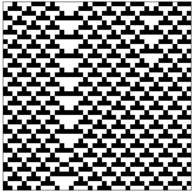

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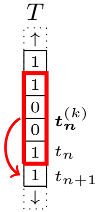

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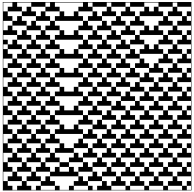

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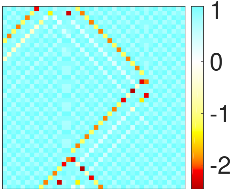

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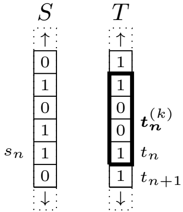

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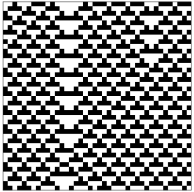

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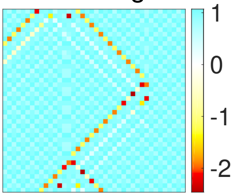

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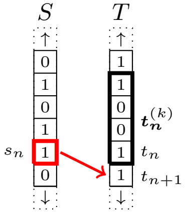

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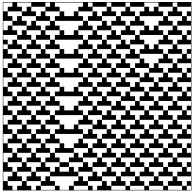

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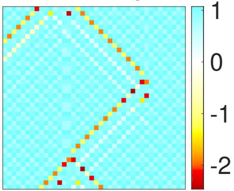

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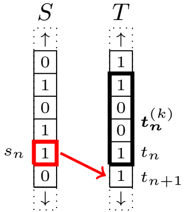

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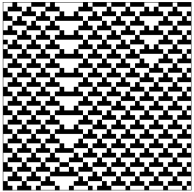

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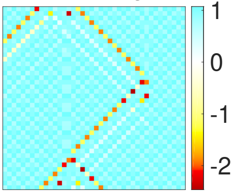
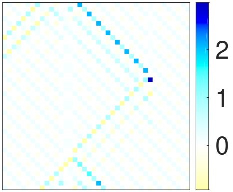


Transfer from left



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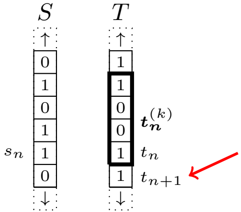

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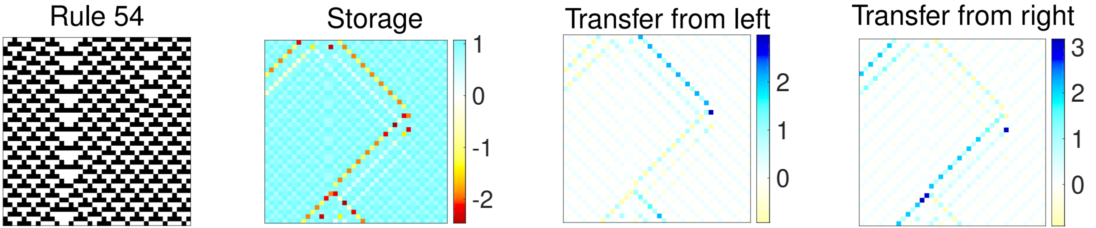

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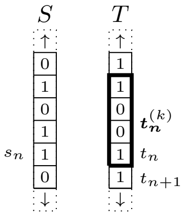
  - modification ← this talk



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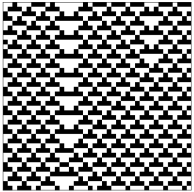

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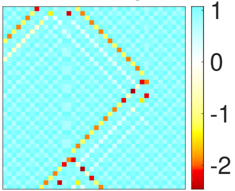

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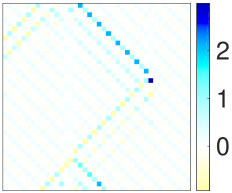

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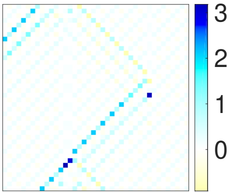

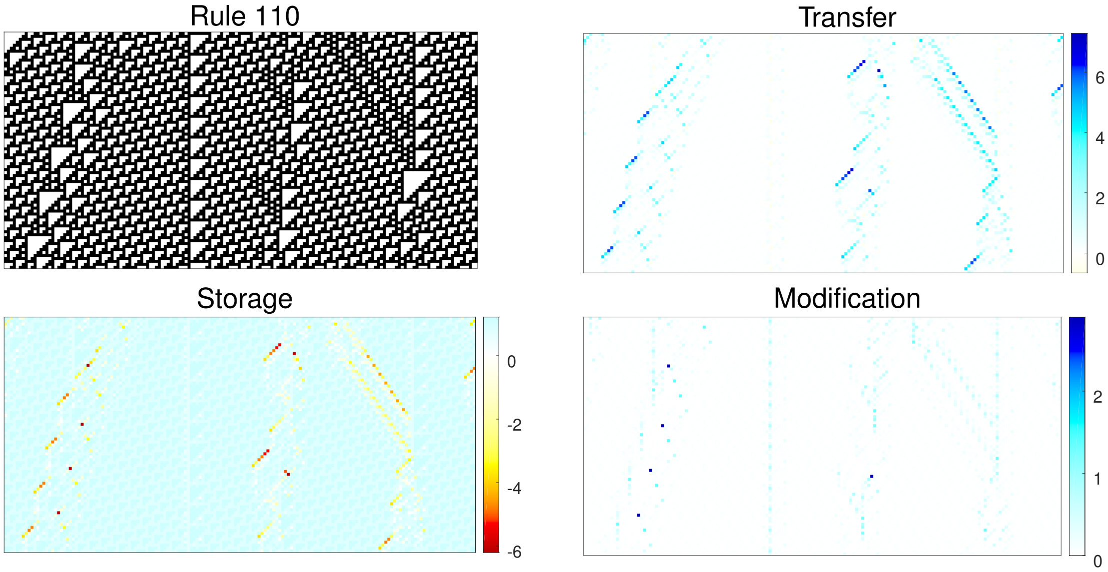

Rule 110
Transfer
6
4
2
0
Storage
0
-2
-4
-6
Modification
2
1
0

How can we quantify information modification?

Lizier et al. (2010)
– Separable information heuristic $\rightarrow$ conflates redundant and synergistic information

Langton (1990)
– An interaction between transmitted and stored information which changes either

Lizier et al. (2013)
– Define non-modified information to antonymically define modified information
– Non-modified information is information identifiable in any **single** source
– Modified information is a non-trivial synthesis of **two or more** sources

Information decomposition

Consider trying to predict T from S_1 and S_2

- Several types of information

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  - **Unique information** $U(S_1 \setminus S_2 \to T)$

Unq

p	s_1	s_2	t
1/4	0	0	0
1/4	0	1	0
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Rdn

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RDN

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XOR

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- Mutual information captures

$$I(T; S_1) = R(T : S_1, S_2) + U(T : S_1 \setminus S_2)$$
$$I(T; S_2) = R(T : S_1, S_2) + U(T : S_2 \setminus S_1)$$

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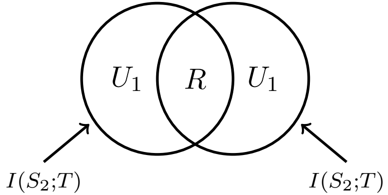

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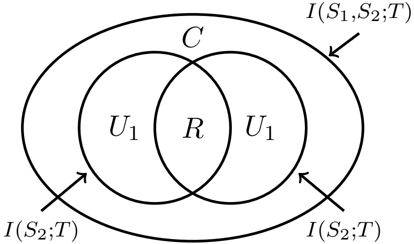

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- Joint mutual information captures

$$I(T;S_1 S_2) = R(S_1,S_2 \to T) + U(S_1\ S_2 \to T) + U(S_2\ S_1 \to T) + C(S_1,S_2 \to T)$$

Partial Information Decomposition (Williams and Beer, 2010)

- Axioms for redundant information
  1. Commutativity
  2. Monotonically decreasing
  3. Self-redundancy (idempotency)

- Yields a redundancy lattice

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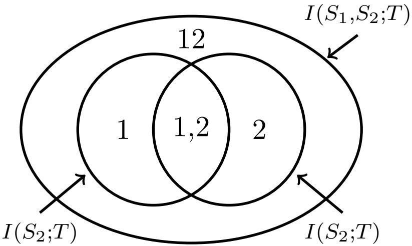

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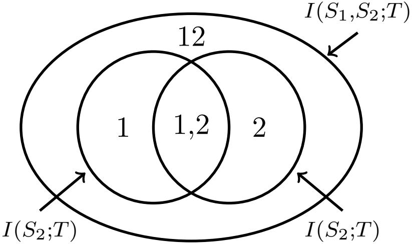

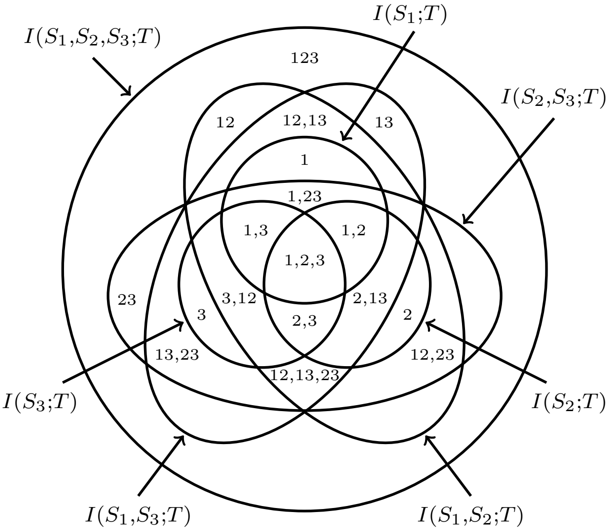

PID is elegant, however...

- Unique evaluation requires a definition of redundant information
  - providing this definition has been a contentious area of research
- Most appoaches do not work for two or more sources (not very useful)
- Information dynamics requires a pointwise information decomposition

entropy

Article

Pointwise Partial Information Decomposition Using the Specificity and Ambiguity Lattices

Conor Finn [1,2,*] and Joseph T. Lizier [1]

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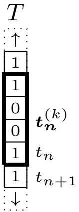
PID and Information Dynamics

Order 1 information

- identifiable in single sources
- non-modified information



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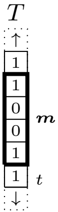

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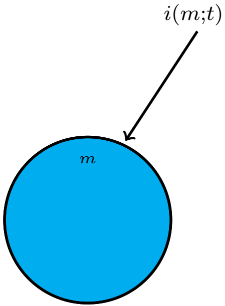

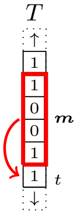

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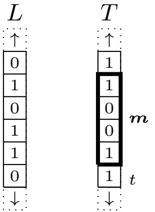

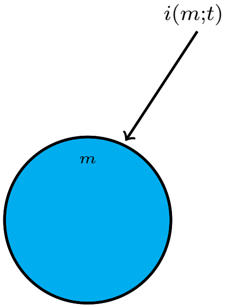

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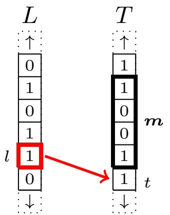

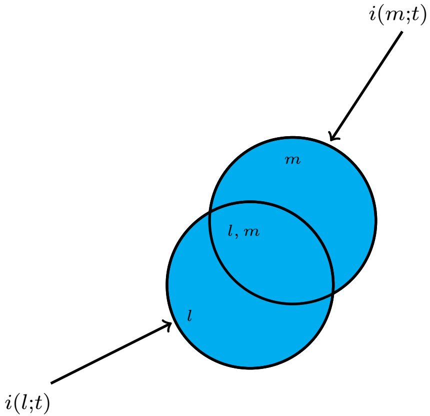

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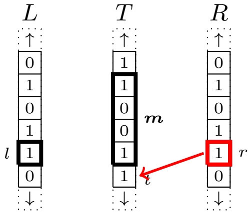

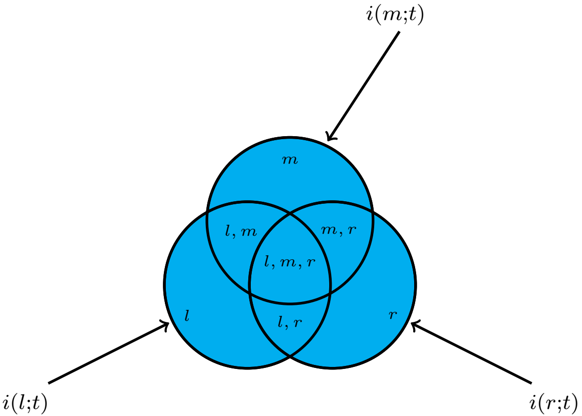

PID and Information Dynamics

Order 1 information

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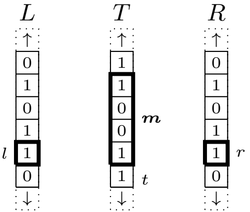
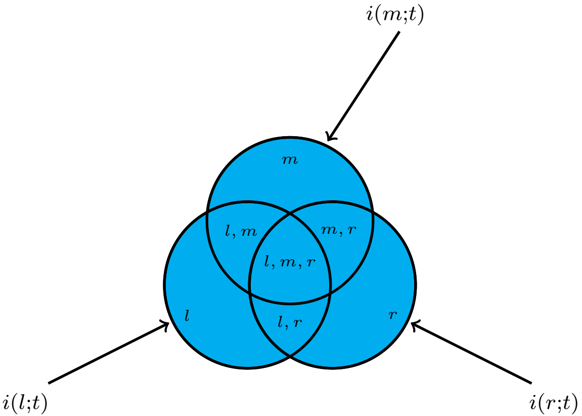
- non-modified information

Order 2 information

- identifiable in pairs of source





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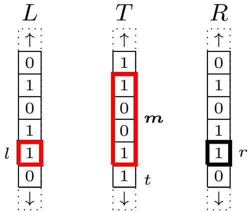
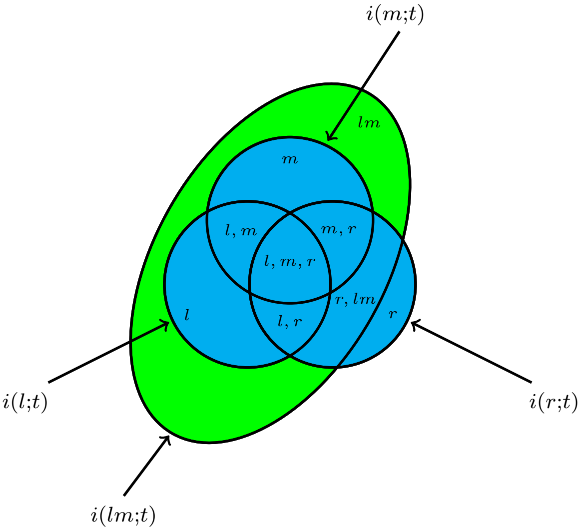
- identifiable in pairs of source





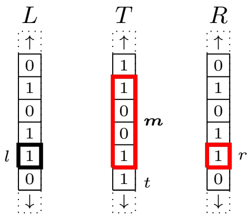
PID and Information Dynamics

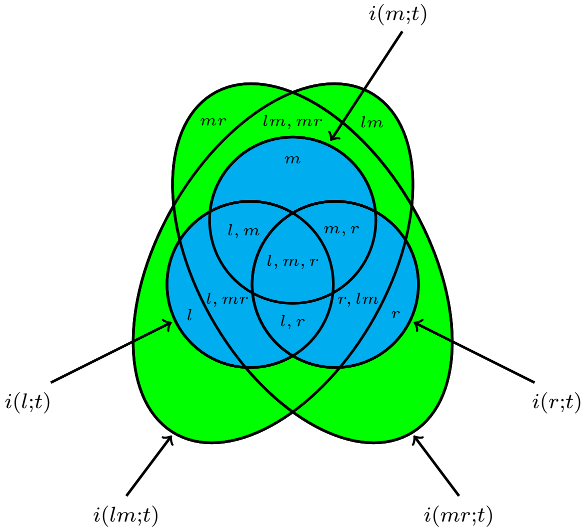
Order 1 information

- identifiable in single sources
- non-modified information

Order 2 information

- identifiable in pairs of source





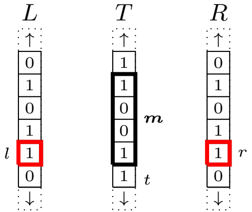
PID and Information Dynamics

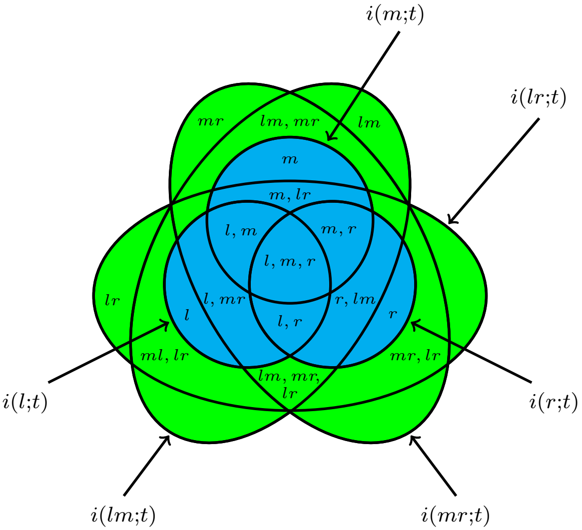
Order 1 information

- identifiable in single sources
- non-modified information

Order 2 information

- identifiable in pairs of source



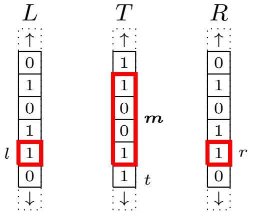


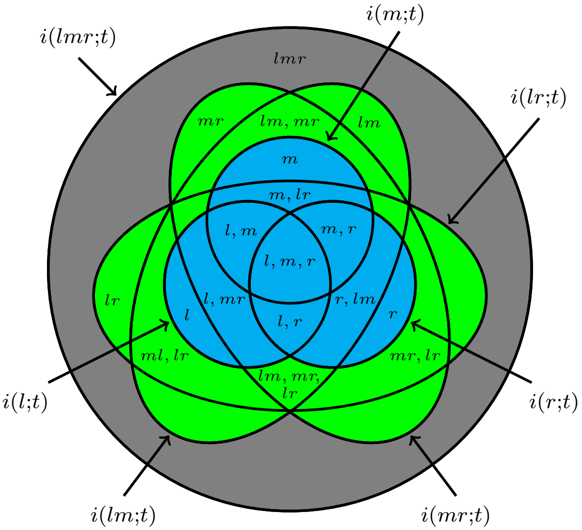
PID and Information Dynamics

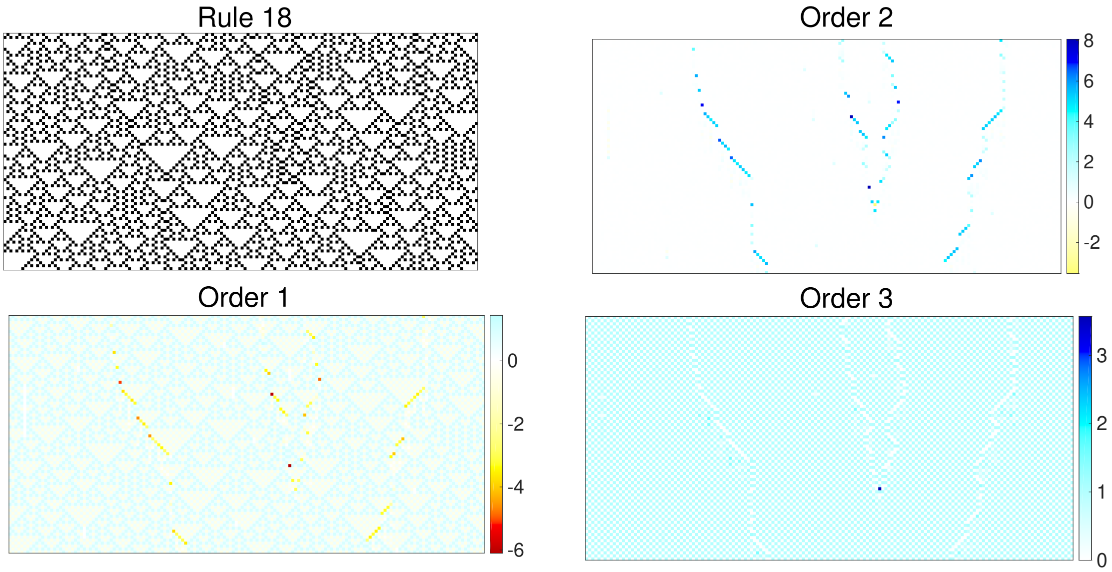
Order 1 information
- identifiable in single sources
- non-modified information

Order 2 information
- identifiable in pairs of source

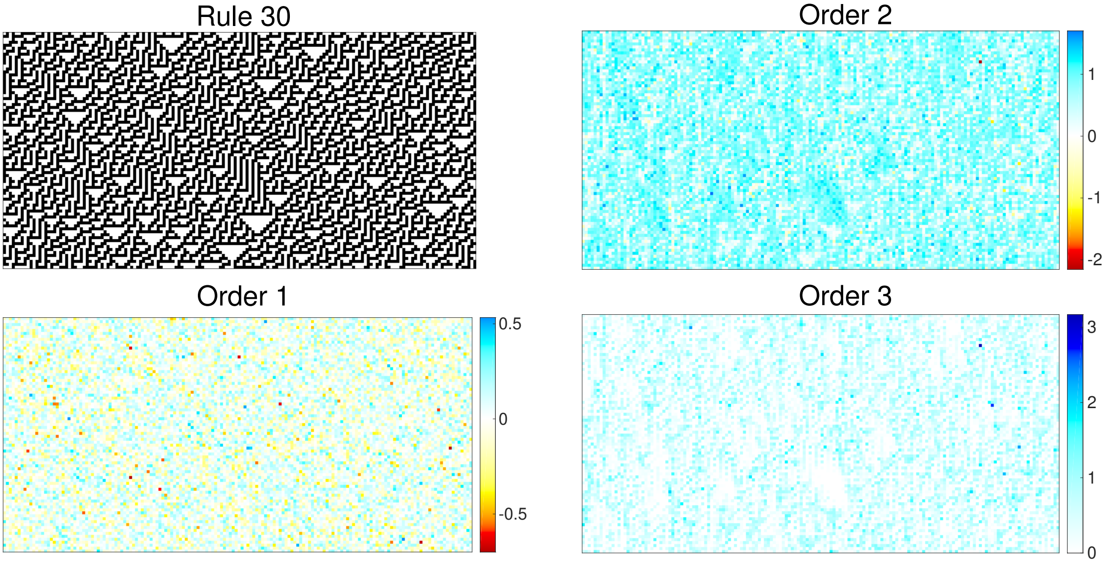
Order 3 information
- identifiable in the triplet







Rule 18
Order 2
8
6
4
2
0
-2
Order 1
0
-2
-4
-6
Order 3
3
2
1
0


Rule 30
Order 2
1
0
-1
-2
Order 1
0.5
0
-0.5
Order 3
3
2
1
0

Questions?

- Full paper on information modification will appear soon
- An extended abstract is available:
  – `https://finnconor.github.io/publications/`
- Measures will soon be released in JIDT:
  – `https://github.com/jlizier/jidt`
- Interested in information decomposition:
  – Information Processing in Complex Systems satellite
  – 12:00 in LHN-TR+04: “Generalised Measures of Multivariate Information Content”

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Joseph T Lizier, Benjamin Flecker, and Paul L Williams. Towards a synergy-based approach to measuring information modification. In *Artificial Life (ALIFE), 2013 IEEE Symposium on*, pages 43–51. IEEE, 2013.

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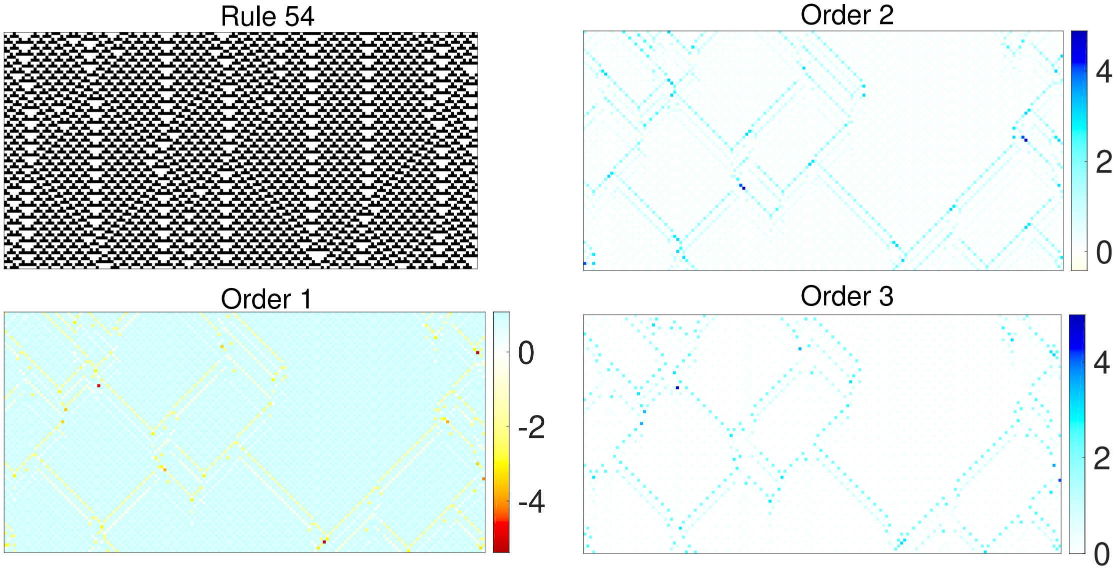
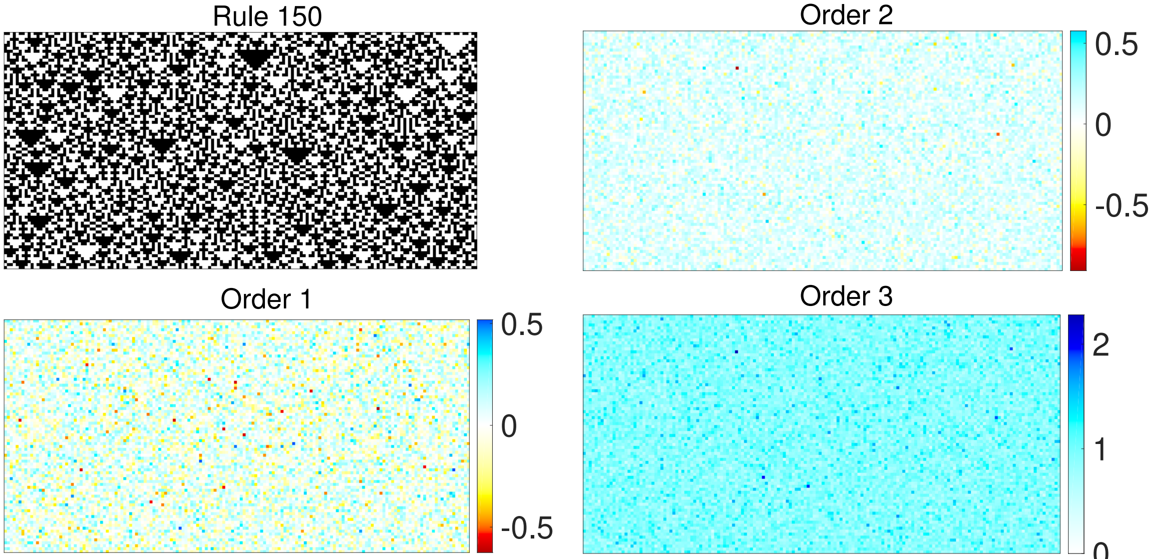
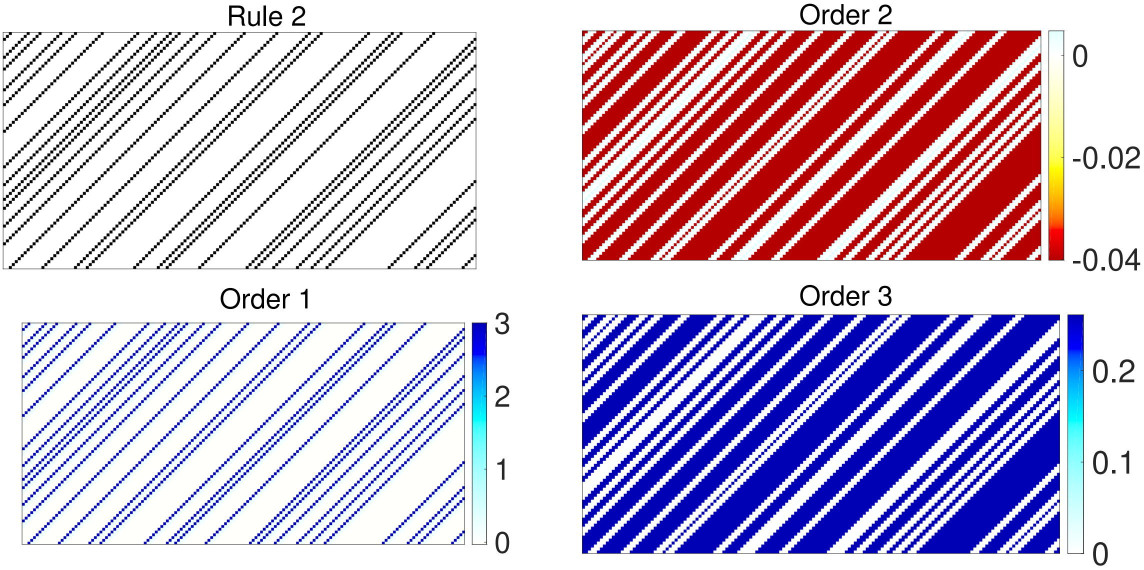
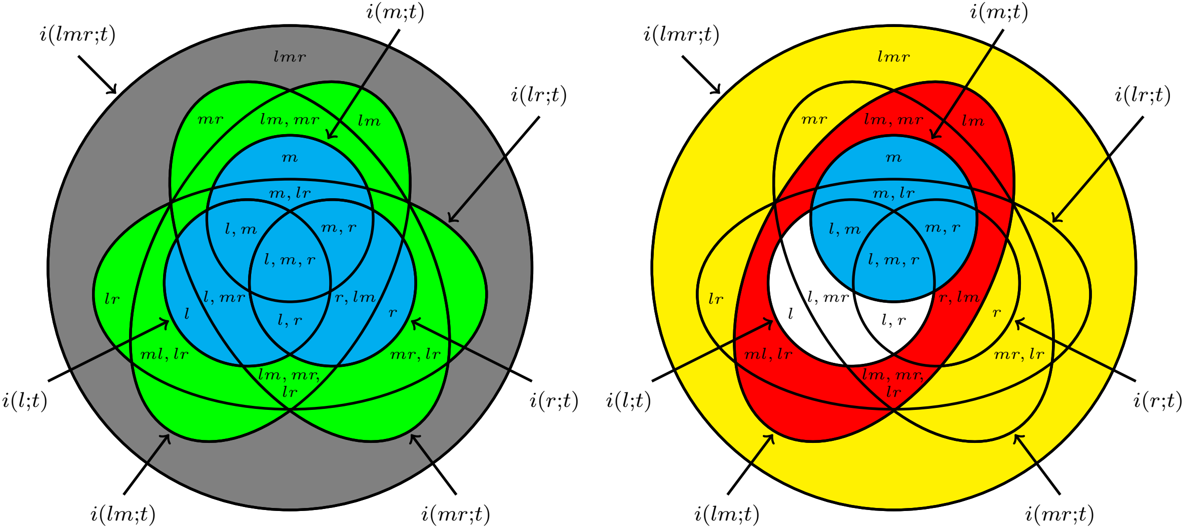

Rule 54
Order 2
4
2
0
Order 1
0
-2
-4
Order 3
4
2
0

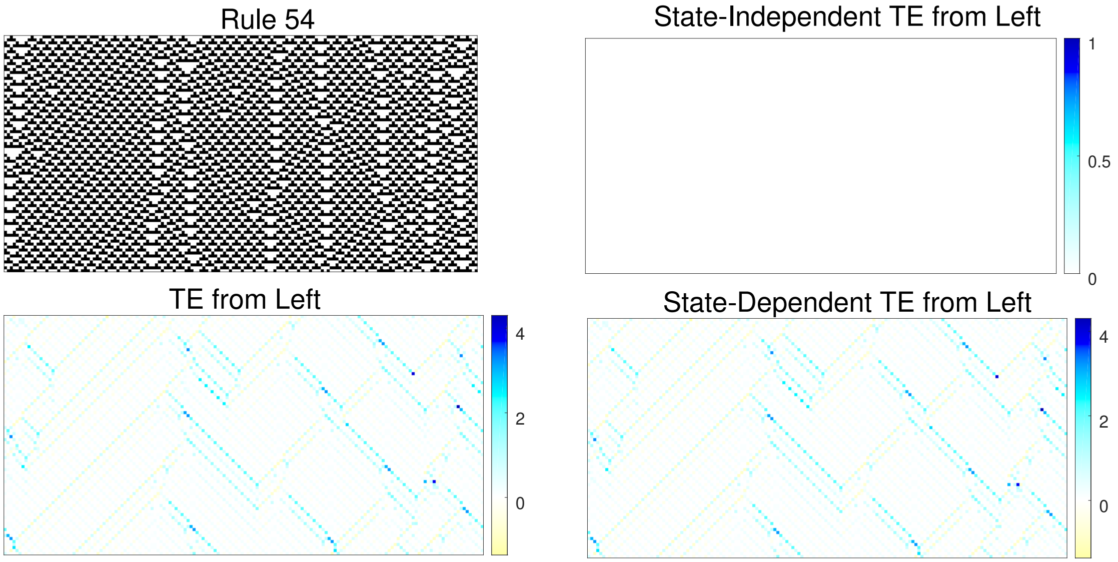


Rule 150
Order 2
0.5
0
-0.5
Order 1
0.5
0
-0.5
Order 3
2
1
0

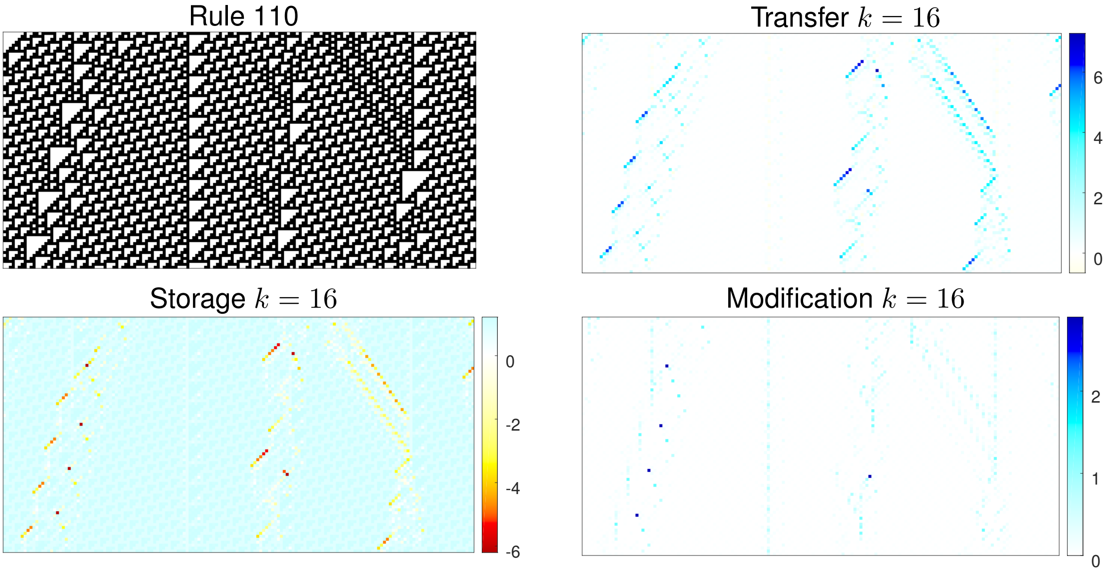


Rule 2
Order 2
0
-0.02
-0.04
Order 1
3
2
1
0
Order 3
0.2
0.1
0

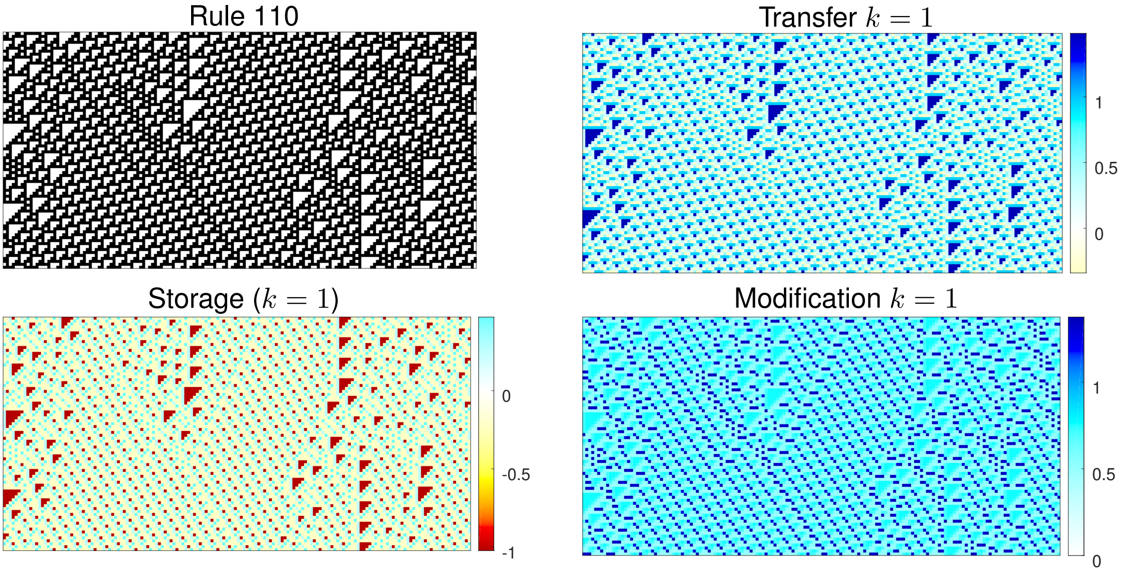

$i(lmr;t)$
$i(m;t)$
$i(lr;t)$
lmr
mr
lm, mr
lm
m
m, lr
l, m
m, r
l, m, r
lr
l, mr
r, lm
l
r
l, r
ml, lr
mr, lr
$lm, mr,$
lr
$i(l;t)$
$i(r;t)$
$i(lm;t)$
$i(mr;t)$
$i(lmr;t)$
$i(m;t)$
$i(lr;t)$
lmr
mr
lm, mr
lm
m
m, lr
l, m
m, r
l, m, r
lr
l, mr
r, lm
l
r
l, r
ml, lr
mr, lr
$lm, mr,$
lr
$i(l;t)$
$i(r;t)$
$i(lm;t)$
$i(mr;t)$



Rule 54
State-Independent TE from Left
1
0.5
0
TE from Left
4
2
0
State-Dependent TE from Left
4
2
0



Rule 110
Transfer $k = 16$
6
4
2
0
Storage $k = 16$
0
-2
-4
-6
Modification $k = 16$
2
1
0


Rule 110
Transfer $k = 1$
1
0.5
0
Storage ($k = 1$)
0
-0.5
-1
Modification $k = 1$
1
0.5
0